1

(a)

Principles 6/10/2010

O = output current in mA
I = input pressure in bar
I_I = interfering input – degrees Celsius
I_M = modifying input – degrees Celsius

(b)

Standard condition :
O [mA] = K.I + a = 3.2 [mA/bar] x I [bar] + 0.8 [mA]
K = 3.2 [mA/bar]
a = 0.8 [mA]
(straight line between the 2 extreme points (1 bar,4.0 mA) and (6 bar, 20 mA))

Non-standard conditions – since slope AND offset are changed the temperature difference of 10 °C can be considered both an interfering and modifying input.

O [mA] = 3.7 [mA/bar] x I [bar] – 0.2 [mA]

3.7 [mA/bar] = K + $K_M I_M$ ➔ $K_M I_M = K_M$ 10 °C = 0.5 [mA/bar] ➔ **K_M = 0.05 [mA/bar/°C]**
-0.2 [mA] = a + $K_I I_I$ ➔ $K_I I_I = K_I$ 10 °C = -1.0 [mA] ➔ **K_I = - 0.1 [mA/°C]**

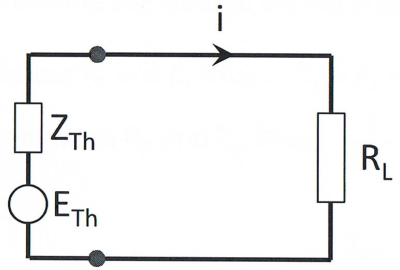
2

(a)

$$P = \frac{V_S^2}{R} = 1 \text{ Watt}$$

$\mathbf{V_S = 10}$ **Volt**

(b)



$Z_{Th}^{-1} = (30\ \Omega)^{-1} + (70\ \Omega)^{-1}$

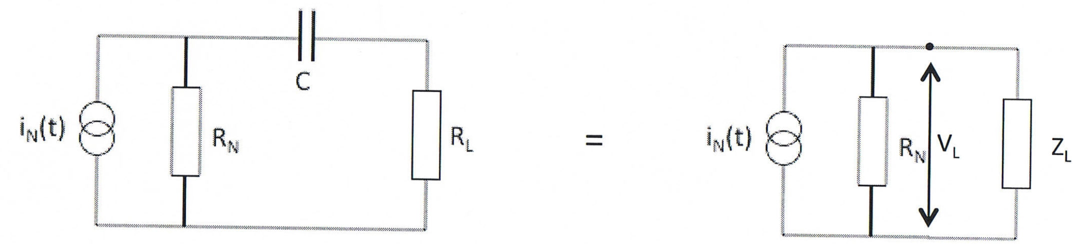
$Z_{Th} = 21\ \Omega$

$E_{Th} = 7\ V$

(c) $V_L = E_{Th} R_L / (Z_{Th} + R_L) = 0.95\ E_{Th}$

$0.95 = R_L / (21\ \Omega + R_L)$

$\mathbf{R_L = 399\ \Omega}$



3.a) **ANALYZE the electrical circuit, as was done for the NORTON equivalent circuit (p. 82-84)**

First method : determine the voltage on the output terminals of the NORTON equivalent circuit

Z_L = series impedances R_L and C, thus : $Z_L = R_L + Z_C = R_L + \frac{1}{sC}$

Z_{TOT} = Parallel impedances R_N and Z_L, thus : $\frac{1}{Z_{TOT}} = \frac{1}{R_N} + \frac{1}{Z_L} = \frac{1}{R_N} + \frac{1}{R_L + \frac{1}{sC}} = \frac{1}{R_N} + \frac{sC}{sCR_L + 1} = \frac{1 + sC(R_N + R_L)}{R_N(sCR_L + 1)}$

$$Z_{TOT} = \frac{R_N(sCR_L + 1)}{1 + sC(R_N + R_L)}$$

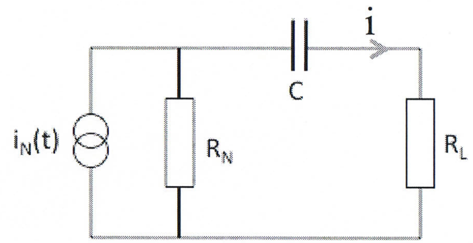
$$V_L = i_N Z_{TOT} = i_N \frac{R_N(sCR_L + 1)}{1 + sC(R_N + R_L)}$$

Voltage over the resistance R_L = $V_{R_L} = V_L \frac{R_L}{R_L + \frac{1}{sC}} = V_L \frac{sCR_L}{sCR_L + 1} = i_N \frac{R_N(sCR_L + 1)}{1 + sC(R_N + R_L)} \frac{sCR_L}{(sCR_L + 1)} = i_N \frac{R_N sCR_L}{1 + sC(R_N + R_L)}$

In the spirit of the "impedance transfer function for the electrical circuit" (p. 86, Bentley), one recognizes easily

the form of a transfer function (V_{RL}/i_N) $\frac{ks}{1 + \tau s}$

3



3.a) **ANALYZE the electrical circuit, as was done for the NORTON equivalent circuit (p. 82-84)**

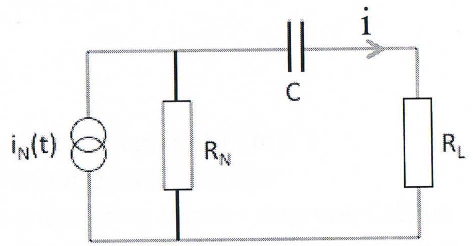
Second method : use the current division rule in the NORTON circuit

$$i = i_N \frac{R_N}{R_N + R_L + \frac{1}{sC}}$$

$$V_{R_L} = iR_L = i_N \frac{R_L R_N}{R_N + R_L + \frac{1}{sC}} = i_N \frac{sCR_L R_N}{sC(R_N + R_L) + 1}$$

again one recognizes the form of G(s) = $\frac{ks}{1+\tau s}$

3



3.b) $k = CR_N R_L$

$\tau = C(R_N + R_L)$

τ = 0.2 seconds

k = 10^4 seconds x Ohm

3.c) 1^{st} order

3.d) $G(s) = \frac{ks}{1+\tau s}$

$$|G(s=j\omega)| = \left|\frac{jk\omega}{1+j\tau\omega}\right| = \left|\frac{k\tau\omega^2 + jk\omega}{1+(\tau\omega)^2}\right| =$$

$$\frac{\sqrt{(k\tau\omega^2)^2 + (k\omega)^2}}{1+(\tau\omega)^2} = k\omega\frac{\sqrt{(\tau\omega)^2+1}}{1+(\tau\omega)^2} = k\omega\frac{1}{\sqrt{1+(\tau\omega)^2}}$$

amplitude of the input sine wave = I
amplitude of the output sine wave = I.|G(s=jω)|
ratio is thus just simply |G(s=jω)|

3.e) $\lim_{\omega\to\infty}|G(s=j\omega)| \approx \frac{k\omega}{\tau\omega} = \frac{k}{\tau}$

3.f) $|G(s=j\omega)| = k\omega\frac{1}{\sqrt{1+(\tau\omega)^2}} = k\omega\frac{1}{\sqrt{1+(\tau\omega)^2}} = \frac{1}{\sqrt{2}}\frac{k}{\tau}$

$\Rightarrow \omega^2\frac{1}{1+(\tau\omega)^2} = \frac{1}{2\tau^2}$

$\Rightarrow 2\tau^2\omega^2 = 1+\tau^2\omega^2$

$\Rightarrow \omega = \frac{1}{\tau}$

bandwidth ranges from 1/τ to infinity

3.g) phase = $\arg(G(j\omega)) = \tan^{-1}\left(\frac{1}{\omega\tau}\right)$

for ω very small the phase goes to 90 degrees
for ω very big the phase goes to zero degrees.

3.h) output $f_0(s) = \frac{ks}{1+\tau s}\cdot\frac{A}{s} = A\frac{k}{1+\tau s}$

where A is the magnitude of the step function
(Norton current i_N = A)

inverse Laplace transform :

$f_o(t) = i_N\frac{k}{\tau}\exp(-t/\tau) = i_N 50k\Omega\exp(-t/0.2\sec)$